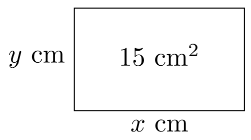
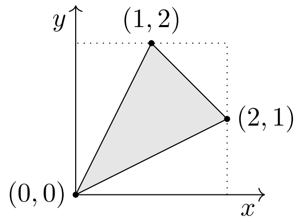
1. A rectangle, with sides length x cm and y cm, has perimeter 16 cm and area 15 cm^2.



   (a) Show that $x + y = 8$.
   (b) Write down a second equation linking x and y.
   (c) Hence, show that $x^2 - 8x + 15 = 0$.
   (d) Solve to find the side lengths of the rectangle.

2. Solve the equation $\dfrac{2x-1}{2x+1} = 3$.

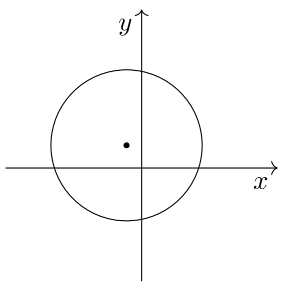
3. A triangle has vertices as depicted.



   (a) Show that the perimeter is $\sqrt{2} + 2\sqrt{5}$.
   (b) By considering the square with dotted edges, or otherwise, find the area of the triangle.

4. Simplify the following, leaving your answers in fully factorised form:
   (a) $a + b \times c - d + d + c \times b - a$,
   (b) $ab(c - d) - bc(a - d) + ad(b - c)$.

5. A circle has equation $x^2 + 4x + y^2 - 6y = 87$.



   (a) By completing the square, express the circle as
   $$(x - a)^2 + (y - b)^2 = r^2.$$
   (b) Hence, write down the centre and the radius.

6. One of the following statements is true; the other is not. Identify and disprove the false statement.
   (a) $x^2y^2 = 0 \implies x^2 = 0$,
   (b) $x^2 = 0 \implies x^2y^2 = 0$.

7. Find the equation of the straight line through the points $(-2, -7)$ and $(1, 2)$, in the form $y = mx + c$.

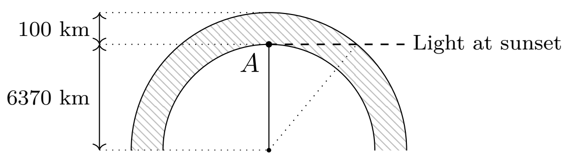
8. Independent events A and B have $\mathbb{P}(A) = 0.4$ and $\mathbb{P}(B) = 0.1$. Using a tree diagram, or otherwise, find the following probabilities:
   (a) $\mathbb{P}(A \cap B)$,
   (b) $\mathbb{P}(A \cup B)$.

9. Simplify the following expression, where $x - a$ and $x - b$ are both positive:
$$\sqrt{\frac{x-a}{x-b}} \times \sqrt{\frac{a-x}{b-x}}.$$

10. This problem was written down on a clay tablet in Babylon, 4000 years ago: "At a non-compound interest rate of $\frac{1}{60}$/month, find the time taken for the initial amount invested to double."

11. Integers a and b are such that their difference is 100 and their product is 5964. By setting up and solving a pair of simultaneous equations, find all possible values of a and b.

12. In this question, model Earth as a sphere of radius 6370 km, and the atmosphere as extending to a constant height of 100 km above the surface. At point A on the Earth's surface, it is sunset, with light arriving from the sun along the dashed line:



   (a) Explain how you know that, at sunset, light arrives at right angles to the Earth's radius.
   (b) Show that, at sunset, sunlight travels through around 11 times more atmosphere on its way to us than it does when the sun is overhead.

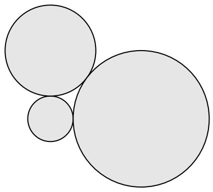
13. By listing them explicitly, show that there are six possible rearrangements of AABB.

14. A quadratic equation is given as $\mathrm{f}(x) = 0$, where $\mathrm{f}(x) = x^2 + kx + 8$, for some constant $k \in \mathbb{Z}$. You are given that $(x - 4)$ is a factor of $\mathrm{f}(x)$.
   (a) Use the factor theorem to find k.
   (b) Hence, solve the quadratic equation.

15. State, with a reason, whether $y = x + 2$ intersects the following lines:
   (a) $y = x - 2$,
   (b) $y = 2 - x$.

16. Solve $\dfrac{(x+3)(x-1)}{(x+2)(x+6)} = 0$.

17. Three circles of radius 1, 2 and 3, with centres at P, Q, R, are all tangent to each other, as shown.



Show that PQR is a right-angled triangle.

18. Simplify $p^{\log_p q} \times q^{\log_q p}$.

19. A parabola passes through the points $(-6, 0)$, $(-4, 0)$ and $(0, 48)$.

(a) Explain why the parabola must have the form $y = a(x+4)(x+6)$.

(b) Determine the value of a.

20. In a statistical study, the elements of the sample have units ms^{-1}. Give the units of

(a) the mean,

(b) the inter-quartile range,

(c) the variance,

(d) the standard deviation.

21. Write down the equation of the locus of points equidistant from $y = 4x + 6$ and $y = 4x + 10$.

22. The variable p is inversely proportional to the square of x and proportional to the square root of y. When $x = 1$, $y = 5$. Find y in terms of x.

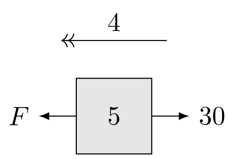
23. Simplify the following, where $n \in \mathbb{N}$, giving your answers in standard form:

(a) $5 \times 10^n + 6 \times 10^n$,

(b) $5 \times 10^n \times 6 \times 10^n$.

24. Sketch the graph $xy = 1$.

25. An object is modelled in the force diagram below, with mass given in kilograms, forces in Newtons, and acceleration in ms^{-2}.



Show that $F = 50$.

26. Two dice are rolled together. By drawing a 6×6 table of the outcomes (a possibility space), find the probability that the sum of the two scores is 8.

27. A linear function is $\text{f}(x) = ax + b$, for $a, b \in \mathbb{R}$. Prove that $\text{f}(k - x) + \text{f}(k + x)$ is independent of x for any $k \in \mathbb{R}$.

28. The inequality $x^2 - 2x - 8 > 0$ is given.

(a) Write down the boundary equation.

(b) Solve the boundary equation.

(c) Sketch $y = x^2 - 2x - 8$.

(d) Hence, determine the set of values of x which satisfy the inequality.

29. The first-principles differentiation of the parabola $y = x^2$ is set up in the following limit:

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}.$$

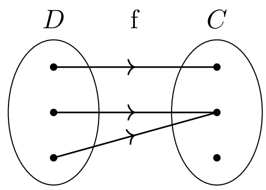
(a) With reference to a sketch, explain why this limit gives the gradient of the curve at the point (x, x^2).

(b) Expand and factorise the numerator.

(c) Hence, show that $\dfrac{dy}{dx} = 2x$.

30. Complete the square to write $4x^2 + 24x + 54$ in the form $a(x+b)^2 + c$, where $a, b, c \in \mathbb{Z}$.

31. A mapping f maps a domain D to a codomain C, according to the following scheme:



State, with a reason, whether $\text{f} : D \mapsto C$ is

(a) a well-defined function,

(b) invertible.

32. Solve the inequality $3 - 2x > 4$, giving your answer in set notation.

33. Three integers are as follows: twice the first is the second, thrice the second is the third, and all three add to eighteen. Find the integers.

34. (a) Calculate $5x^2 - 12x + 4\,\big|_{x=2}$.

(b) Hence, show that $(x - 2)$ is a linear factor of the expression $5x^2 - 12x + 4$, giving the name of the theorem you use.

35. Variables a, b, c are linked by $a = b^2$ and $b = c+1$. Express c in terms of a.

36. Sketch $x = -y^2$.

37. A car of mass 800 kg sets off from rest along a straight horizontal road. The driving force is 1000 N, and there is a resistance force of 400 N.
    (a) Draw a force diagram.
    (b) Determine the acceleration.
    (c) Find the displacement after 20 seconds.

38. Three coins are tossed.
    (a) List the possibility space.
    (b) Find the probability that tossing three coins yields exactly two heads.

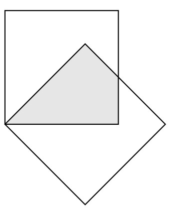
39. The graph $y = x^2 - x - 6$ crosses the x axis twice.
    (a) Find the x coordinates of these intercepts.
    (b) Find $\frac{dy}{dx}$.
    (c) Show that, at its x intercepts, the gradient of the parabola is ± 5.
    (d) Explain why the gradients of a quadratic graph $y = \mathrm{f}(x)$ at its x axis intercepts must always be of the form $\pm k$.

40. Rearrange the following to make x the subject:

$$y = \frac{x}{x+a}.$$

41. A student solves $9x^2 - 1 > 0$, and gets $\frac{1}{3} < x < -\frac{1}{3}$. Explain the notational error and correct it.

42. Two squares of side length 1 are drawn with one vertex in common, and with an edge of one along the diagonal of the other, as depicted.



Show that the area of the shaded kite is $\sqrt{2} - 1$.

43. Solve, to 3sf, the equation $3^{x+1} = 7$.

44. The equation $ax^2 + bx + y^2 + 4y = c$ is a circle which passes through the origin and is centred on a point satisfying $y = x$. Find the constants a, b, c.

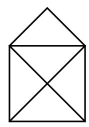
45. Sketch the graph $x^2 = y^2$.

46. Two students are calculating the displacement, over a given period of time, of a particle whose velocity is $v = \frac{1}{2}t + 3$, using areas under a velocity-time graph. The first student calculates the area using the formula for the area of a trapezium; the second sets up the following (correct) integral:

$$\Delta x = \int_{t=4}^{t=6} \tfrac{1}{2}t + 3\, dt = 11.$$

    (a) Sketch a velocity-time graph, and shade the region whose area is Δx.
    (b) Use the first student's method to verify that $\Delta x = 11$.

47. A simple game consists of drawing the following shape, whose base has length 1 and which consists of horizontal, vertical and 45° lines, without taking the pen off the paper.



Find the total length of the line drawn.

48. Solve the equation $(x^2 + 1)^7(3x - 2) = 0$.

49. An object of mass m kg has exactly two forces acting on it, whose magnitudes are $2m$ and $3m$ N. Give the minimum and maximum possible values for the magnitude of the acceleration.

50. Give, with a reason, the formula for the exterior angle θ, defined in radians, of a regular n-gon.

51. By completing the square twice, show that the equation $x^2 + 6x + y^2 - 3y = 0$ defines a circle, and determine its centre and exact radius.

52. Find the following sums:

(a) $\displaystyle\sum_{r=1}^{10} 1$, (b) $\displaystyle\sum_{r=0}^{n} 1$, (c) $\displaystyle\sum_{r=1}^{n} a$.

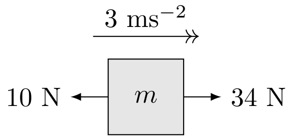
53. By first finding the roots of the equation using an analytic method such as the quadratic formula, explain why the decimal search method would be likely to fail to find a root of $42x^2 - 71x + 30 = 0$.

54. A bird has velocity $12\mathbf{i} - 5\mathbf{j}$ ms^{-1}. Find its speed.

55. An inequality is given as $-x^2 - 6x \geq 0$.
    (a) Solve the boundary equation.
    (b) Sketch the graph $y = -x^2 - 6x$.
    (c) Hence, solve the inequality, giving your answer in interval set notation.

56. Explain why the discriminant $\Delta = b^2 - 4ac$ gives the number of real roots of a quadratic equation.

57. An object is modelled as below:



Solve to find the mass m.

58. If $\mathrm{f}(x) = 4x\left(1 - x^2\right)$, find $\mathrm{f}'(x)$.

59. Write down the number of ways of rearranging:

(a) ABC,

(b) AAB.

60. Express $1\frac{1}{2}$ right angles in radians.

61. Points A, B have position vectors $\mathbf{a}$, $\mathbf{b}$, relative to an origin O. M is the midpoint of AB. Give the following vectors in terms of $\mathbf{a}$ and $\mathbf{b}$:

(a) $\overrightarrow{AB}$,

(b) $\overrightarrow{OM}$,

62. The equation $(x-p)(x-q)(x-\frac{1}{2}q) = 0$ has roots $x = 2, 3, 4$. Find p and q.

63. In any given year, the probability of my birthday falling on a Tuesday is $\frac{1}{7}$. Explain, using formal language, what is wrong with this statement: "The probability of my birthday falling on a Tuesday in two consecutive years is $\frac{1}{7} \times \frac{1}{7} = \frac{1}{49}$."

64. Solve the following quadratic in x^2:

$$x^4 - x^2 - 72 = 0.$$

65. By dropping a suitable perpendicular, prove the area formula $A_{\triangle} = \frac{1}{2}ab\sin C$ for triangles.

66. You are given that invertible functions f and g have $\mathrm{f}(1) = 2$, $\mathrm{f}(2) = 3$, $\mathrm{g}(2) = 3$ and $\mathrm{g}(3) = 4$. Write down the values of

(a) $\mathrm{gf}(2)$,

(b) $\mathrm{f}^{-1}\mathrm{g}^{-1}(4)$,

(c) $\mathrm{g}^{-1}\mathrm{f}^2(1)$.

67. Evaluate $\log_a a^2 + \log_b b^3$.

68. By starting with the right-hand side, prove that

$$\frac{1}{pq} \equiv \frac{1}{p(p+q)} + \frac{1}{q(p+q)}.$$

69. Solve the equation $|2x - 1| = 9$.

70. A curve is defined, for constants a, b, c, d, by

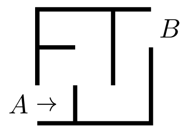
$$y = \frac{(x+a)(x-b)}{(x+c)(x-d)}.$$

The numbers $\pm a, \pm b, \pm c, \pm d$ are distinct. Write down the equations of the vertical asymptotes.

71. A ship sets out from port A. Having travelled 24.1 nautical miles, it corrects course by 20°, arriving at port B after a further 18.7 nautical miles. Find the extra distance it travelled above the minimum.

72. "The line $x = 1$ is tangent to the curve $y = x^2 - x$." True or false?

73. A robot is navigating the maze below, starting at point A and moving in the direction shown. When it hits a wall, it turns 90° left or right, choosing the direction at random.



Find the probability that it navigates to B along the shortest route available to it.

74. The quadratic function $\mathrm{f}(x) = ax^2 + 4x + 15$, where a is a constant, has range $[7, \infty)$.

(a) Explain how you know that a must be positive.

(b) By differentiating, or otherwise, show that the minimum value of $\mathrm{f}(x)$ is at $x = -\frac{2}{a}$.

(c) Hence, show that $a = \frac{1}{2}$.

75. In one case, write down the derivative with respect to x; in the other, explain why no derivative exists.

(a) $x = 4$,

(b) $y = 4$.

76. Solve $(3x - 7)(x^2 + 1)(x^2 - 4) = 0$.

77. Evaluate $\Big[x^2 + x + 1\Big]_0^4$.

78. A sample of data $\{x_i\}$ is coded according to the formula $y_i = ax_i + b$. Write down the mean $\bar{y}$ and variance s_y^2 of the new set $\{y_i\}$, in terms of $a, b, \bar{x}$ and s_x^2.

79. Give counterexamples to the following statements:

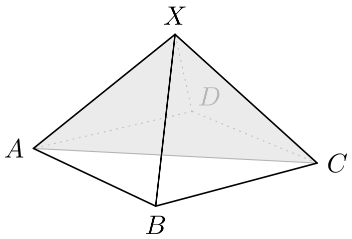
(a) $x \in \mathbb{Z} \implies x \in \mathbb{N}$,

(b) $x \in \mathbb{Q} \implies x \in \mathbb{Z}$,

(c) $x \in \mathbb{R} \implies x \in \mathbb{Q}$.

80. Rationalise the denominator of $\dfrac{3}{\sqrt{7}-2}$.

81. The square-based pyramid shown below is formed of eight edges of unit length.



Show that triangle AXC is right-angled.

82. Disprove the following statement: "If the decimal expansion of a number does not terminate, then the number is irrational."

83. Verify that the function $\mathrm{f}(x) = \sin \frac{a}{b}x$ and its derivative $\mathrm{f}'(x) = \frac{a}{b}\cos\frac{a}{b}x$ satisfy the equation

$$\big(a\,\mathrm{f}(x)\big)^2 + \big(b\,\mathrm{f}'(x)\big)^2 = a^2.$$

84. Find the probability that, when a coin is tossed three times, all three tosses show the same result.

85. Variables x and y take the following values:

x	1	3	6
y	0	4	9

Show that the relationship cannot be linear.

86. Solve the equation $(2^x+1)(2^x-8) = 0$.

87. Two runners are planning a 1 km race, and want to choose a time handicap for the faster runner. A runs at 5 ms^{-1}, while B runs at 4 ms^{-1}. Find the time delay for A which would have them finish the race together.

88. Simplify $\sqrt{\dfrac{x-p}{x-q}} \div \sqrt{\dfrac{q-x}{p-x}}$.

89. Determine which of the points $(3,3)$ and $(5,1)$ is closer to the point $(1,0)$.

90. Simplify the following expressions:

(a) $\log_a \sqrt[3]{a}$,

(b) $\log_{a^2} a$,

(c) $\log_{a^3} a^2$.

91. If $2x+3y=5$, find $\frac{dy}{dx}$.

92. The function $\mathrm{g}(x) = 6x^2 - x - 35$ is defined over the real numbers.

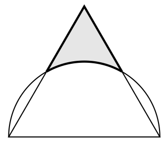
(a) Solve $\mathrm{g}(x) = 0$,

(b) Sketch $y = \mathrm{g}(x)$,

(c) Hence, give the solution set of $\mathrm{g}(x) \leq 0$.

93. Give 2.6 radians to the nearest degree.

94. An arrowhead design is constructed from a semicircle and an equilateral triangle of side length l. The arrowhead has perimeter P.



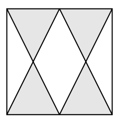
Show that $P = l(1+\frac{1}{3}\pi)$.

95. A straight line has parametric vector equation

$$\mathbf{r} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} t,$$

where t is a parameter allowed to take any value. Find the Cartesian equation of the line, giving your answer in the form $ax+by=c$, for $a,b,c \in \mathbb{Z}$.

96. A design is drawn on a square, connecting vertices and midpoints of edges as below.



Find the fraction of the total area that is shaded.

97. Two sets of data are to be combined. For $i = 1,2$, they contain n_i data, with mean $\bar{x}_i$. Determine a formula for the mean of the combined set of data.

98. Prove that the area of a trapezium is $\frac{1}{2}(a+b)h$.

99. Give counterexamples to the following:

(a) $a < b \implies a^2 < b^2$,

(b) $a < b \implies \frac{1}{a} < \frac{1}{b}$.

100. Two forces of magnitudes 6 and 8 Newtons act on an object of mass 10 kg. Find the acceleration if those forces act

(a) in the same direction,

(b) perpendicular to each other,

(c) in opposite directions.

——— End of 1st Hundred ———